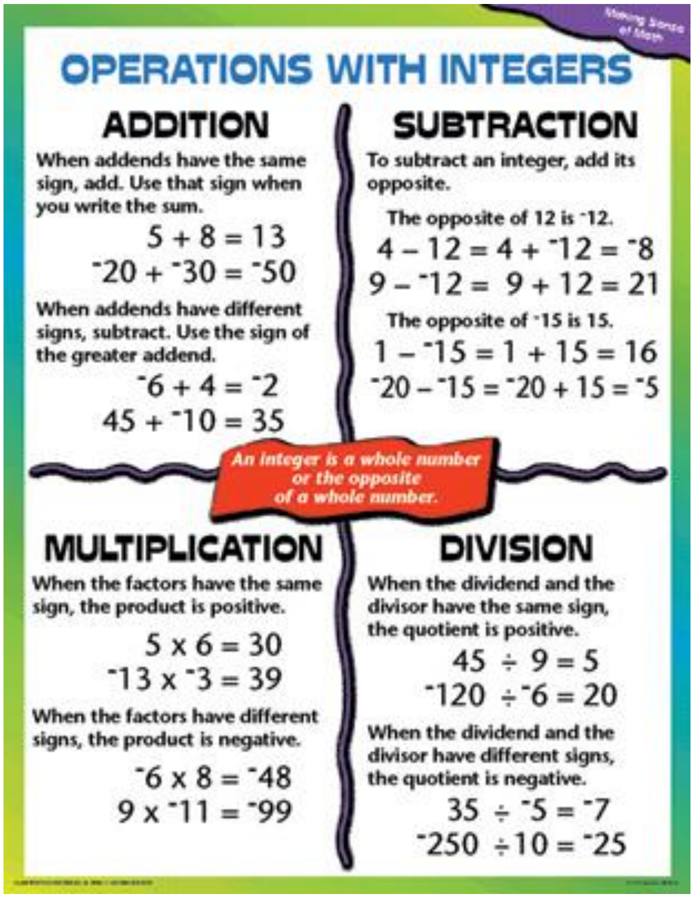
# MCAS MATH

# Grade 10 Review Packet

Southwick Regional School
Mathematics Department

# <u>Rules of Integer Operations</u>




| Concept Summary for Solving Systems of Equations | |
|---|---|
| **Method** | **Best Time to Use It** |
| Graphing | To estimate solutions, since graphing usually does not give an exact solution |
| Substitution | If one of the variables in either equation has a coefficient of 1 or -1 |
| Elimination Using Addition | If one of the variables has opposite coefficients in the two equations |
| Elimination Using Subtraction | If one of the variables has the same coefficient in the two equations |
| Elimination Using Multiplication | If none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations |

**Steps to solving Systems of Equations by Substitution:**

$x + 3y = 6$
$2x + 8y = -12$

1. Isolate a variable in one of the equations. (Either y = or x =).

$x + 3y = 6$
$x = 6 - 3y$

2. Substitute the isolated variable in the other equation.

$2x + 8y = -12$
$2(6 - 3y) + 8y = -12$

3. This will result in an equation with one variable. Solve the equation.

$12 - 6y + 8y = -12$
$2y = -24$
$y = -12$

4. Substitute the solution from step 3 into another equation to solve for the other variable.

$x + 3(-12) = 6$
$x = 42$

5. Recommended: Check the solution.

$42 = 6 - 3(-12)$

| **6x + 5y =4**<br>**6x - 7y = -20** | **Solve the system using Elimination** |
|---|---|
| $-1(6x + 5y) = -1(4)$<br>$-6x - 5y = -4$ | Multiply one equation by negative 1 to try to ELIMINATE one of the variables |
| $-6x - 5y = -4$<br>$+\ 6x - 7y = -20$<br>$-12y = -24$ | Eliminate the y-variable by adding the two equations together |
| $\frac{-12y}{-12} = \frac{-24}{-12}$<br>$y = 2$ | Divide by -12 to solve for x |
| $6x + 5(2) = 4$ | Plug the y-value into any previous equation to solve for x |
| $6x + 10 = 4$<br>$-10 \quad -10$<br>$\frac{6x}{6} = \frac{-6}{6}$<br>$x = -1$ | Solve for x by subtracting 10 and then dividing by 6 |
| $(x, y) = (-1, 2)$ | Solution |

**Point-slope form of a line**

$y - y_1 = m(x - x_1)$

**Slope-Intercept form of a line**

$y = mx + b$

EX: **Find the equation of the line that has slope $m = 4$ and passes through the point (–1, –6).**

$y - y_1 = m(x - x_1)$

| | |
|---|---|
| $y - (-6) = (4)(x - (-1))$ | Plug in the values of the slope and point |
| $\mathbf{y + 6 = 4(x + 1)}$ | Simplify. This is now in point-slope form. |
| $y + 6 = 4x + 4$ | To put into y=mx+b, solve for y |
| $y = 4x + 4 - 6$ | |
| $\mathbf{y = 4x - 2}$ | Same equation in slope-intercept form |

**The Midpoint formula :** $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

When given 2 points, add the x-coordinates and divide by 2, and add the y-coordinates and divide by 2. This gives you the coordinates of the midpoint of the segment that is between the given points.

EX: $R$ is the midpoint between $Q(-9, -1)$ and $T(-3, 7)$. Find its coordinates.

$$R = \left( \frac{-9 + -3}{2}, \frac{-1 + 7}{2} \right)$$

$$R = \left( \frac{-12}{2}, \frac{6}{2} \right)$$

$$R = (-6, 3)$$

**The Distance Formula**

$(x_1, y_1)$ $(x_2, y_2)$

$$D = \sqrt{\left( (x_2 - x_1)^2 + (y_2 - y_1)^2 \right)}$$

To find the distance between two points, use the distance formula. EX:

Find the distance between the two points.

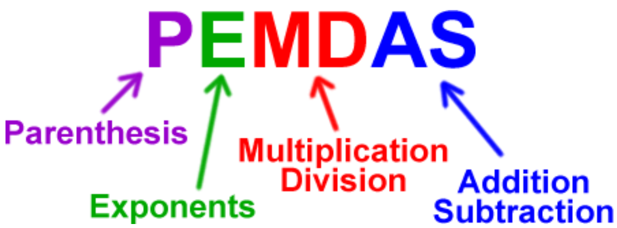
$(-5, 0), (-2, 2)$

$$d = \sqrt{(-2 + 5)^2 + (2 - 0)^2}$$

$$d = \sqrt{(3)^2 + (2)^2}$$

$$d = \sqrt{9 + 4}$$

$$d = \sqrt{13}$$




Evaluate the expression:

**9/3 + 2(9+10) – 8 + 4•3**

| | |
|---|---|
| 9/3+2(9+10)-8 + 4•3 | Parenthesis |
| 9/3 +2(19) – 8 + 4•3 | Division (comes 1st) |
| 3+2(19) – 8 + 4•3 | Multiplication (comes 2nd) |
| 3+ 38 – 8 + 4•3 | Multiplication (comes 3rd) |
| 3+38 – 8 + 12 | Addition (comes 1st) |
| 41 – 8 + 12 | Subtraction |
| 33 + 12 | Final Addition |
| **45** | **45 is the final answer.** |

# Sequences

To find any term in an **arithmetic sequence**, use

$$a_n = a_1 + (n-1)d$$

where $a_1$ is the first term of the sequence,
$d$ is the common difference, $n$ is the number of the term to find.

| | |
|---|---|
| Find the $10^{th}$ term of the sequence<br>3, 5, 7, 9, ...<br><br>*the common difference is 2 because when you subtract each number from the number ahead of it you get 2 every time. | **3**. $n = 10$; $a_1 = 3$, $d = 2$<br><br>$a_n = a_1 + (n-1)d$<br>$a_{10} = 3 + (10-1)2$<br>$a_{10} = 21$<br>The tenth term is 21. |

To find any term in a **geometric sequence**, use

$$a_n = a_1(r)^{n-1}$$

where $a_n$ is the $n^{th}$ term of the sequence (the term you are looking for), $a_1$ is the first term, n is the number of the term you are finding, and r is the common ratio.

EX: What's the 23rd term of this sequence? $\frac{1}{9}, -\frac{1}{3}, 1, -3, 9, \ldots$

Find the ratio:

$$r = a_2 \div a_1 = -\frac{1}{3} \div \frac{1}{9} = -\frac{1}{3} \cdot \frac{9}{1} = -3$$

So…

$$a_{23} = \frac{1}{9}(-3)^{22}$$
$$= -10{,}460{,}353{,}203$$

Yikes!

By the way, these ( ) are really important if your ratio is negative! So, be careful!

## Distributive Property

Multiply what's outside the () by everything inside the ()

EX:

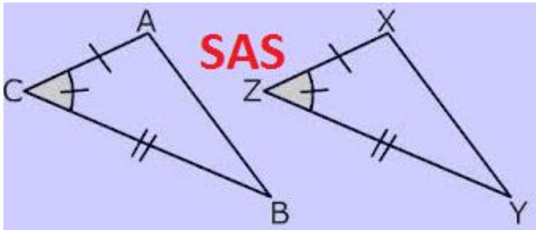
$$5\,(x + 2) = 5 \bullet x + 5 \bullet 2$$

*watch your signs $5(x-2) = 5 \bullet x - 5 \bullet 2$

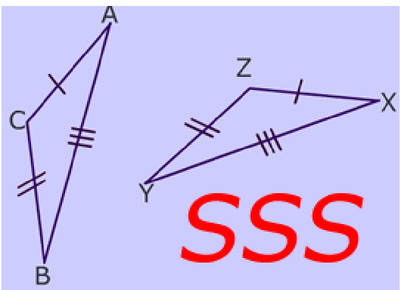
## Algebraic Properties

| | |
|---|---|
| Commutative Property of Addition<br>$a + b = b + a$ | $2 + 3 = 3 + 2$ |
| Commutative Property of Multiplication<br>$a \bullet b = b \bullet a$ | $2 \bullet (3) = 3 \bullet (2)$ |
| Associative Property of Addition<br>$a + (b + c) = (a + b) + c$ | $2 + (3 + 4) = (2 + 3) + 4$ |
| Associative Property of Multiplication<br>$a \bullet (b \bullet c) = (a \bullet b) \bullet c$ | $2 \bullet (3 \bullet 4) = (2 \bullet 3) \bullet 4$ |
| Distributive Property<br>$a \bullet (b + c) = a \bullet b + a \bullet c$ | $2 \bullet (3 + 4) = 2 \bullet 3 + 2 \bullet 4$ |
| Additive Identity Property<br>$a + 0 = a$ | $3 + 0 = 3$ |
| Multiplicative Identity Property<br>$a \bullet 1 = a$ | $3 \bullet 1 = 3$ |
| Additive Inverse Property<br>$a + (-a) = 0$ | $3 + (-3) = 0$ |
| Multiplicative Inverse Property<br>$a \cdot \left(\frac{1}{a}\right) = 1$<br>**Note:** *a* cannot = 0 | $3 \cdot \left(\frac{1}{3}\right) = 1$ |
| Zero Property<br>$a \bullet 0 = 0$ | $5 \bullet 0 = 0$ |

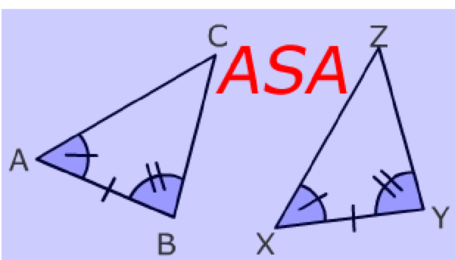
## Proving Triangle Congruence



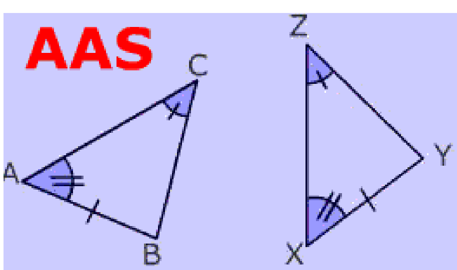

Two sides and the included angle (the one in between) of one triangle are congruent to the corresponding parts of the other triangle.




All three sides of one triangle are congruent to the corresponding three sides of the other triangle.

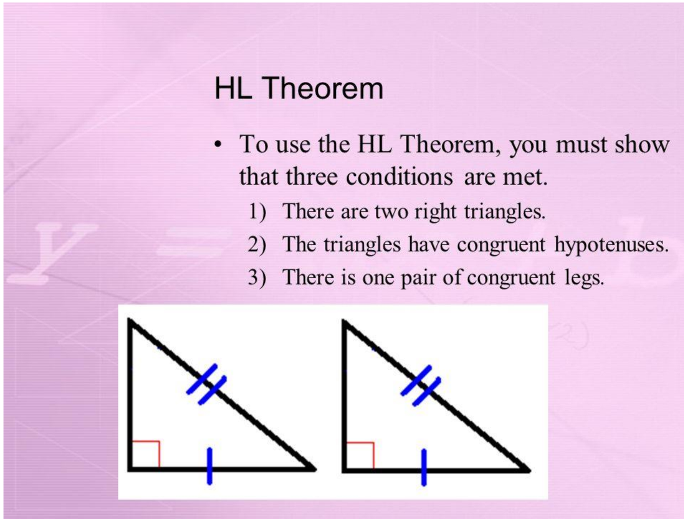



Two angles and the included side of one triangle are congruent to the corresponding parts of the other triangle.

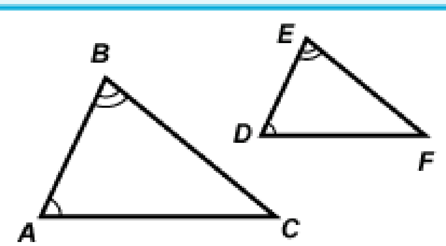
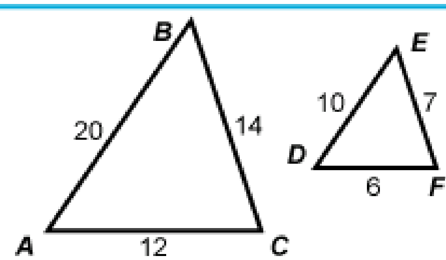
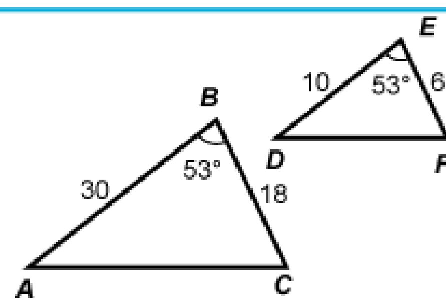



Two angles and the non-included side (the one not in between) of one triangle are congruent to the corresponding parts of the other triangle.

## Congruence by the hypotenuse-leg Theorem




## Triangle Similarity

| Similarity Shortcuts | | |
|---|---|---|
| **Postulate or Theorem** | **Explanation** | **Example** |
| **AA Postulate** (Angle-Angle) | Two triangles are similar if they have two pairs of congruent angles. |  |
| **SSS Theorem** (Side-Side-Side) | Two triangles are similar if they have three pairs of proportional sides. |  |
| **SAS Theorem** (Side-Angle-Side) | Two triangles are similar if they have two pairs of proportional sides with a congruent included angle. |  |

## Finding the x- and y-intercepts

The x-intercept of a line is the point at which the line crosses the x axis. ( i.e. where the y value equals 0 )

**$x$-intercept = ( $x$, 0 )**

The y-intercept of a line is the point at which the line crosses the y axis. ( i.e. where the x value equals 0 )

**$y$-intercept = ( 0, $y$ )**

EX

Find the $x$ and $y$ intercepts of the equation $3x + 4y = 12$.

**Solution**

To find the $x$-intercept, set $y = 0$ and solve for $x$.

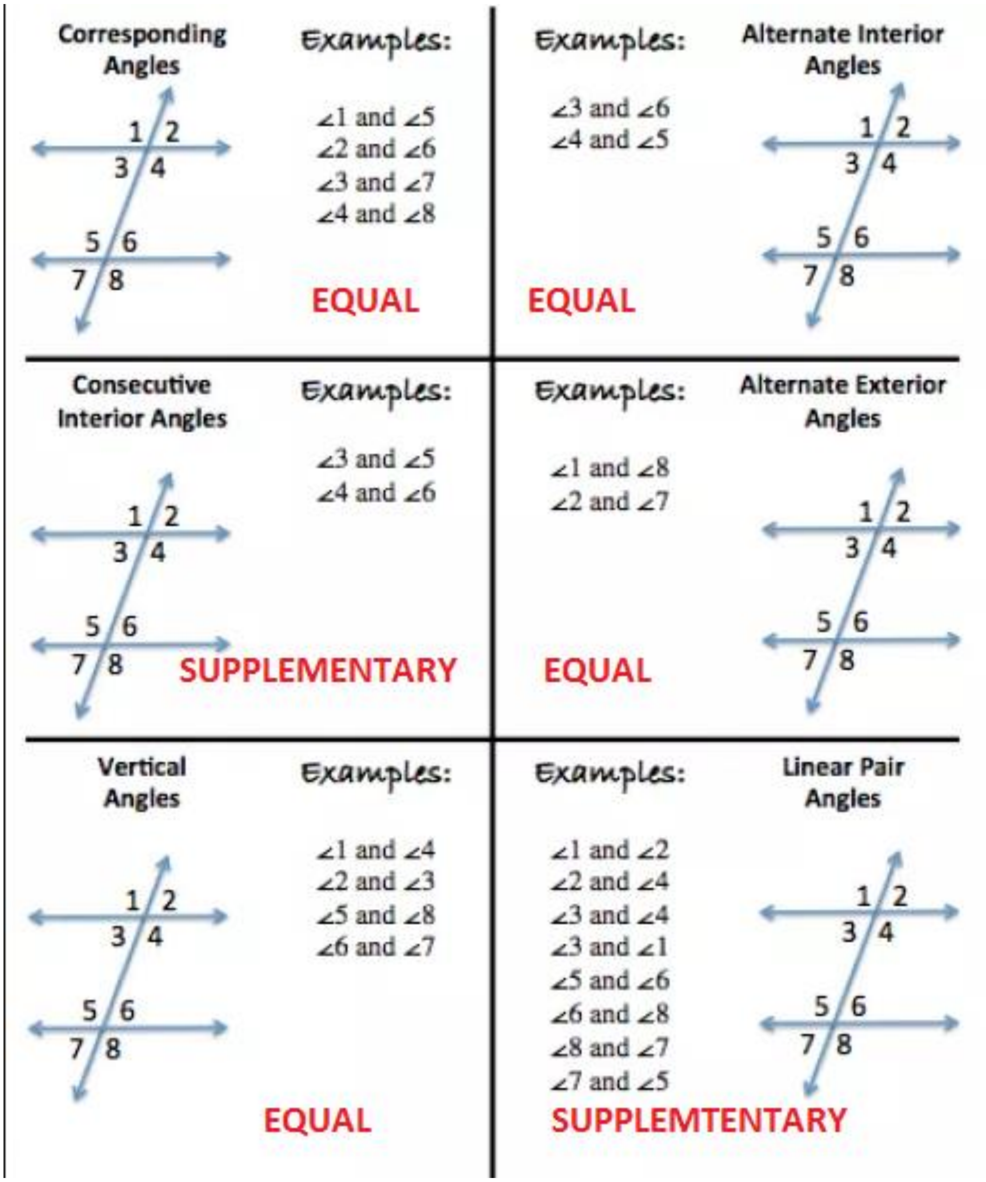
$3x + 4( 0 ) = 12$
$3x + 0 = 12$
$3x = 12$
$x = 12/3$
$x = 4$

To find the $y$-intercept, set $x = 0$ and solve for $y$.

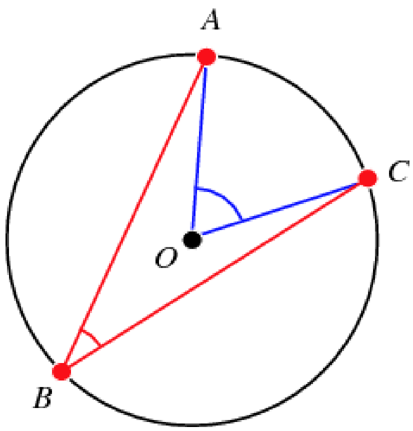
$3( 0 ) + 4y = 12$
$0 + 4y = 12$
$4y = 12$
$y = 12/4$
$y = 3$

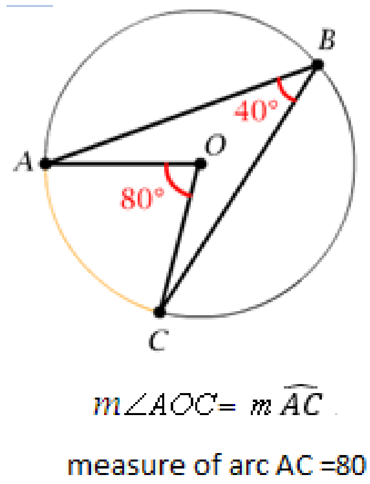
Therefore, the $x$-intercept is ( 4, 0 ) and the $y$-intercept is ( 0, 3 ).

Angles formed by parallel lines cut by a transversal



Central and Inscribed Angles

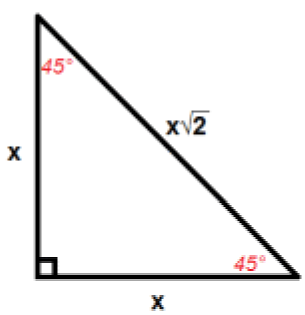


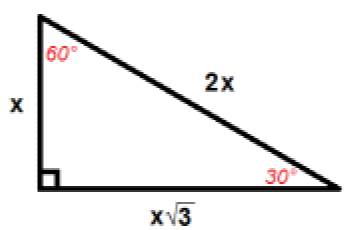

Inscribed angle ( $\angle ABC$ ) is half the measure of its intercepted arc (Arc AC) and central angle ( $\angle AOC$ ) is the same measure as its intercepted arc (also Arc AC).

*Total degree measure of a circle = $360^\circ$

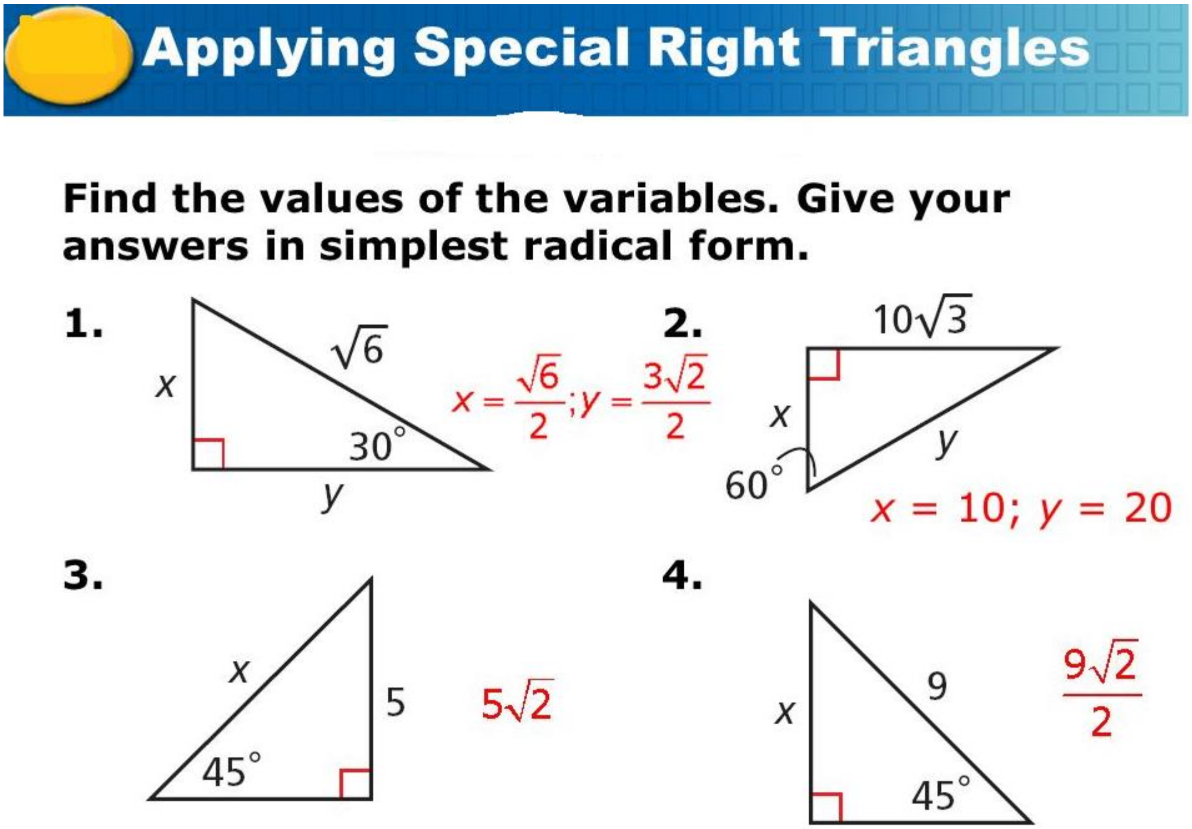
**Special Right Triangles**




45° - 45° - 90° Triangle

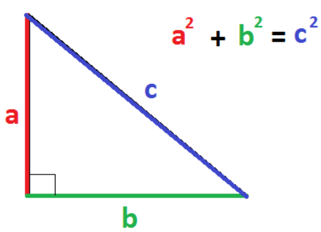



30° - 60° - 90° Triangle

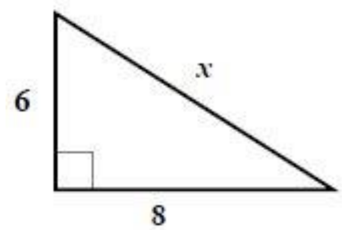


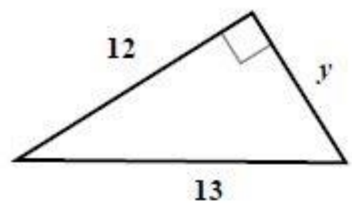

**Pythagorean Theorem**

Use the Pythagorean Theorem to find missing side lengths of a right triangle.



Examples:




$$6^2 + 8^2 = x^2$$
$$36 + 64 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = \sqrt{x^2}$$
$$\boxed{x = 10}$$

$$12^2 + y^2 = 13^2$$
$$144 + y^2 = 169$$
$$y^2 = 25$$
$$\sqrt{y^2} = \sqrt{25}$$
$$\boxed{y = 5}$$

**Approximating Square Roots:** Numbers that are not perfect squares do not have integer square roots. You can use perfect squares to approximate the square root of a number that is not a perfect square.

**Step 1:** Locate the closest perfect squares (one smaller, one bigger).
**Step 2:** Decide which is actually closer to the desired radicand.

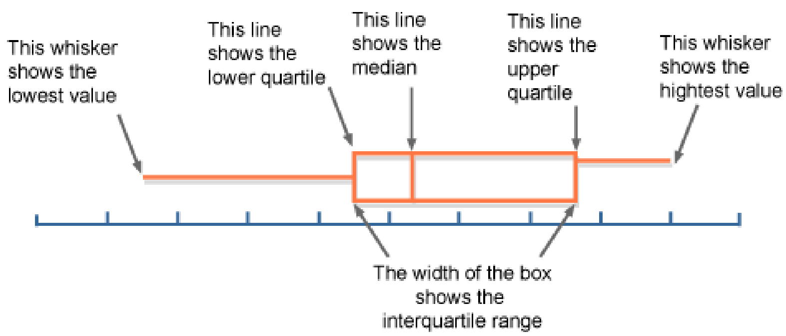
**Example:** Estimate $\sqrt{52}$.
Since 52 is *not* a perfect square, we need to find two perfect squares that are close to 52. They are:

**49 and 64** because $\sqrt{49} < \sqrt{52} < \sqrt{64}$ or $7 < \sqrt{52} < 8$.

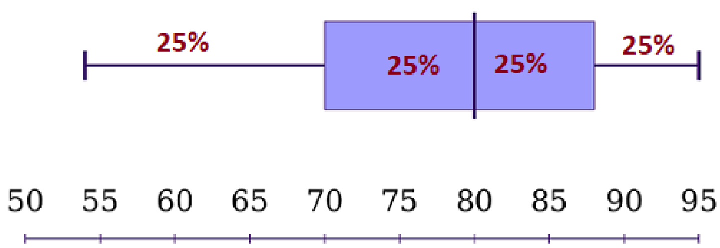
So, $\sqrt{52}$ lies between 7 and 8.
And since 52 is closer to 49, we know that $\sqrt{52}$ is closer to 7.

## Making and reading box and whisker plots

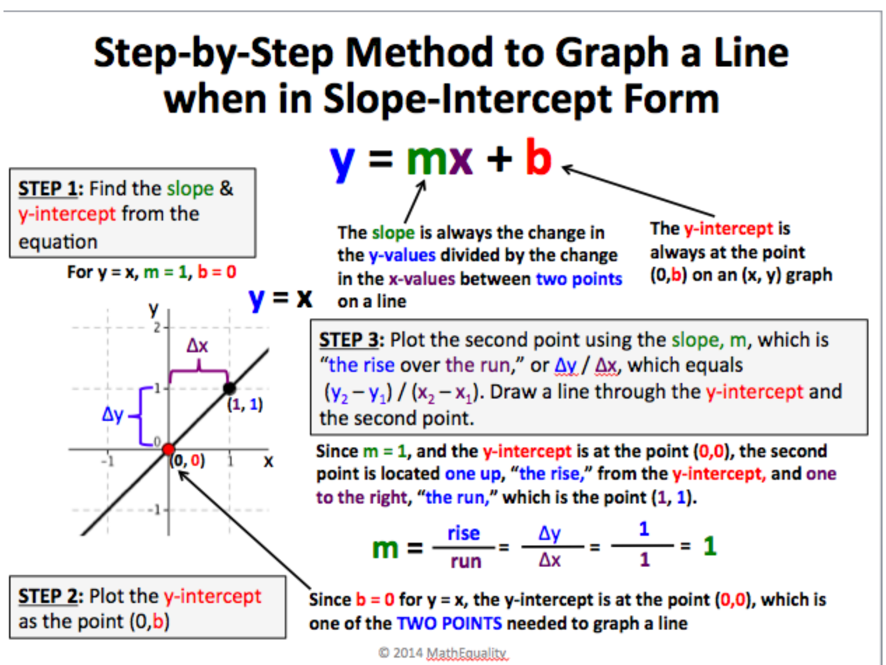
To **create a box-and-whisker plot**, we start by ordering our data (that is, putting the values) in numerical order, if they aren't ordered already. Then we find the median of our data. The median divides the data into two halves. To divide the data into quarters, we then find the medians of these two halves.




A boxplot is a way to show a five number summary in a chart. The main part of the chart (the "box") shows where the middle portion of the data is: the interquartile range. At the ends of the box, you" find the first quartile (the 25% mark) and the third quartile (the 75% mark). The far left of the chart (at the end of the left "whisker") is the minimum (the smallest number in the set) and the far right is the maximum (the largest number in the set). Finally, the median is represented by a vertical bar in the center of the box.

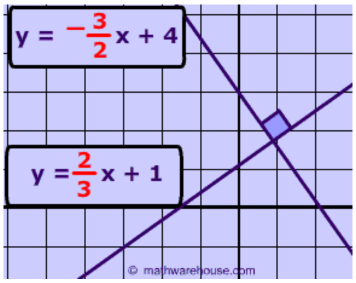


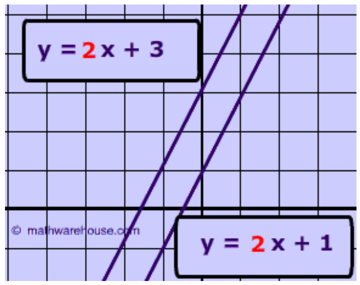
| VOCABULARY | DEFINITION | EXAMPLE |
|---|---|---|
| RANGE | **The difference between the least and greatest values in the set of numbers.** | 2,5,3,6,9,8<br>**9 – 2 = 7**<br>**Range = 7** |
| MEAN | **The sum of all the items, divided by the number of items in the set. Also called the average.** | 2,4,3,6<br><br>**2 + 4 + 3 + 6 = 15**<br>**15 ÷ 4 = 3.75**<br>**Mean = 3.75** |
| MEDIAN | **The middle value when the data are in numerical order. If there are two numbers in the middle, find the mean (average) of those two numbers.** | 2,4,3,6,8,5<br>**2,3,4,5,6,8**<br>**4+5=9**<br>**9÷2=4.5**<br>**4.5 = median**<br><br>2,4,3,6,8,5,7<br>**2,3,4,5,6,7,8**<br>**5 = median** |
| MODE | **The value or values that occurs most often in a set of data.** | 4,5,3,4,3,2,4,6<br>4 = mode |



Slopes of perpendicular and parallel lines

- Perpendicular lines have opposite reciprocal slopes (their product is -1)
- Parallel lines have the same slope




## Factoring Trinomials

**Factor.** $x^2 + 2x - 24$

**Step 1**: List all pairs of numbers that multiply to equal the constant, -24. *(To get -24, one number must be positive and one negative.)*

$-24 = 1 \bullet -24, \ -1 \bullet 24$
$= 2 \bullet -12, \ -2 \bullet 12$
$= 3 \bullet -8, \ -3 \bullet 8$
$= 4 \bullet -6, \ -4 \bullet 6$

**Step 2**: Which pair adds up to 2?

**Step 3**: Write the binomial factors.

$x^2 + 2x - 24 = (x - 4)(x + 6)$

**when the leading coefficient is not 1:**

$ax^2 + bx + c$

If the trinomial has a leading co-efficient that is not just a GCF to be factored out, use a process called **DECOMPOSITION**

$2x^2 + 7x + 5$ — We need to find two numbers whose product is 10 and sum is 7 - 5 and 2.

$2x^2 + 2x + 5x + 7$ — Order of middle terms does not matter. Factor each half separately.

$2x(x+1) + 5(x+1)$ — We now have a binomial with a common binomial factor.

$(x+1)(2x+5)$

**Factoring by Grouping** If you have 4 terms, the process above may still work.

$x^3 + 2x^2 + 8x + 16$
$x^2(x+2) + 8(x+2)$
$(x+2)(x^2+8)$

**Factor Using Grouping**

Group pairs of terms that have common factors.

$2xy + 7x - 2y - 7$
$2xy - 2y + 7x - 7$
$2y(x - 1) + 7(x - 1)$
$(2y + 7)(x - 1)$

The terms must be rearranged in pairs that have a common factor.

What common factor do the first two terms have? **2y**
What common factor do the second two terms have? **7**

Use the distributive property to write the expression in factored form.